COMBINING COVARIATIONAL REASONING AND CAUSALITY TO CONCEPTUALIZE FEEDBACK LOOPS

COMBINANDO EL PENSAMIENTO COVARIACIONAL Y LA CAUSALIDAD PARA CONCEPTUALIZAR BUCLES DE RETROALIMENTACIÓN

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This paper introduces two theoretical constructs, open-loop covariation and closed-loop covariation, that combine covariational reasoning and causality to characterize the way that three preservice mathematics teachers conceptualize a feedback loop relationship in a mathematical task related to climate change. The study's results suggest that the preservice teachers' reasoning about feedback loop between quantities involved the ability to conceive closed-loop covariation, which in this study was characterized by two cognitive realizations: (i) the conception of simultaneous change and (ii) the recognition of circular causality. These realizations, at least for the participants, appeared to be independent from one another. The theoretical distinction between open- and closed-loop covariation could inform instructional strategies to develop students' ability to think about and model feedback loops.

Keywords: Cognition, Integrated STEM / STEAM, Modeling, Sustainability.

Purpose of The Study

Authors from disciplines as diverse as biology, chemistry, engineering, economy, and mathematics have proposed that current global, complex, politically charged, socio-scientific issues (universal income, evolution, pandemic and vaccines, climate change, etc.) require STEM professionals to understand them as complex systems (Ghosh, 2017; Orgill et al., 2019; Renert, 2011; Richmond, 1997; Roychoudhury et al., 2017; Schuler et al., 2018). This holistic perspective, known as systems thinking, focuses on understanding phenomena in terms of relationships, connectedness, and context. Systems thinking complements the analytic or reductionist perspective commonly used in STEM and STEM education fields. An important distinction between these perspectives involves causality; while the reductionist perspective focuses on linear, direct cause-and-effect relationships, systems thinking involves identifying complex causality relationships. An important type of these relationships are feedback loops, or a "succession of cause-effect relations that start and end with the same variable. It constitutes a circular causality, only meaningful dynamically, over time" (Barlas, 2002, p. 1147).

Mathematics represents a powerful way to make sense of relationships, which can be understood as two (or more) quantities changing together over time. Feedback loops, therefore, can be seen as two (or more) quantities changing simultaneously in a way such that, the first quantity *causes* the second quantity to change, and that change *causes* the first quantity to change again, and so on. In particular, I believe that combining covariational reasoning with the notion of causality can provide insights into how students can understand feedback loop relationships in mathematics.

In this paper, I combine covariational reasoning and causality to introduce two theoretical constructs, *open-loop covariation* and *closed-loop covariation*, which can characterize the way that three preservice teachers conceptualize a feedback loop relationship in a mathematical task

related to climate change. The constructs also have the potential to address a gap in the literature focusing on covariational reasoning which have a tendency to consider only unidirectional implications of change and leave underexplored the role of real-world causality. I also discuss possible implications for mathematics learning and teaching.

Conceptual Framework

Covariational Reasoning and the Multiplicative Object

Covariational reasoning finds one of its earliest definitions in the work of Saldanha and Thompson (1998), who defined it as follows:

> Someone holding in mind a sustained image of two quantities' values (magnitudes) simultaneously. It entails coupling the two quantities, so that, in one's understanding, a multiplicative object is formed of the two. As a multiplicative object, one tracks either quantity's value with the immediate, explicit, and persistent realization that, at every moment, the other quantity also has a value … An operative image of covariation is one in which a person imagines both quantities having been tracked for some duration, with the entailing correspondence being an emergent property of the image. (pp. 298-299)

The *multiplicative object* in their definition is analogous to the logical conjunction "and" that joins or units two propositions to produce one proposition that is true if and only if both of the constituent propositions are true. In the case of covariation, the multiplicative object joins the corresponding values of two covarying quantities so that the student "mentally unites their attributes to make a new attribute that is, simultaneously, one and the other" (Thompson et al., 2017, p. 96). This multiplicative object supports the student's ability to conceptualize two (or more) quantities changing simultaneously and interdependently.

Open- and Closed-Loop Covariation

Covariational reasoning support the conceptualization of two quantities changing simultaneously. However, simultaneity may not be enough to conceptualize feedback loop structures in a system. I propose that the way causality is conceived may also play an important role in conceptualizing a feedback loop relationship between two covarying quantities. To address this distinction, I introduced two constructs: *open-loop covariation* (OLC) and *closed-loop covariation* (CLC).

Let's consider the filling bottle problem where water is being pour into the bottle at a constant rate, increasing the volume of water, V, and the water height, h, in the bottle. The multiplicative object allows one to visualize V and h changing simultaneously over time so that there exist a pair $(V(t), h(t))$ for any t in some interval of conceptual time. One imagines that if V changes, so does h, and if h changes, so does V; they change together. The multiplicative object, thus, support a student's ability to conceptualize simultaneous change between two quantities.

However, the multiplicative object does not provide an answer to the questions of *how* and *why* a change in V results in a change in h, or vice versa. In this paper, I use the term causality as a way of describing how and why the state of a dynamic process changes as time goes on (Sauer, 2010). The volume of water V, or the space taken by the water in the bottle, is growing since more and more water is entering the bottle. One also observes (or imagines) that the water height, h, is increasing as water enters the bottle; that is, h grows as V grows. One could say that the growing volume of water in the bottle <u>causes</u> the water height to increase; the more water enters the bottle, the more the water height increases. This is a description of how and why a

change in V causes a change in h. It is important to point out that thinking of causality in the opposite direction may not be as natural or intuitive; mathematically, changing the value of h would change the value of V, but it is hard to imagine that, in a real-world context, an increase in h would cause more water to enter the bottle. I use causality in that second "real-world" sense and consider the covariation of h and V as OLC, which shows *simultaneity* (V and h change together) and only *linear causality* (a change in V causes a change in h, and may not be as intuitive to say that a change in h causes a change in V).

The conceptualization of a feedback loop structure between covarying quantities is only possible when there exists circular causality between those quantities. For instance, consider a simple predator-prey model relationship in which P is the number of predators in a region and N is the number of preys in the same region. We assume that predation (magnitude of P) is the only factor affecting N and that prey availability (magnitude of N) is the only factor affecting P. With those assumptions, P increases when N increases because an increase in availability of prey produces prosperity for predators, who can reproduce more. However, when P increases passed certain value, N starts to decrease because an increase in predator means that more preys would die. When N decreases passed certain threshold value, P would start to decrease as well because predators do not have enough food to sustain their population. As P decreases passed certain value, N becomes to increase again because there are less predators and less preys get eaten. This is an example of CLC, which shows *simultaneity* (P and N change together) and *circular causality* (changes in N cause changes in P, which in turn cause changes in N again and so on). Conceptualizing circular causality, as illustrated by the predators-prey model, seems to be more cognitively demanding than conceptualizing linear causality (Ghosh, 2017; Hokayen et al., 2015; Roberts, 1978; Wellmanns & Schmiemann, 2022).

Methodology

This paper is part of a larger study that investigated how PSTs make sense of some elemental mathematics behind modeling climate change. Three secondary PSTs —hereafter Jodi, Pam, and Kris— enrolled in a mathematics education program at a large Southeastern university participated in the larger study. These PSTs had completed Calculus I and II and an Intro to Higher Mathematics course and were completing a Math Modeling for Teachers course by the time the larger study took place. The PSTs were asked to complete an original sequence of mathematical tasks while participating in individual, task-based interviews (Goldin, 2000). In this paper, I focus on the PSTs' responses to the Energy Balance task.

The Energy Balance (EB) Task

An *energy balance model* describes the continuous heat exchange between the sun, the planet's surface, and the atmosphere (Figure 1a). The planet's surface is warmed by a fraction of the sun's radiation (S). As the surface's temperature increases, it radiates heat towards the atmosphere (R). The majority of it (B) is absorbed by *greenhouse gases* (GHG), which rises the atmosphere's temperature. As it warms up, the atmosphere radiates a fraction of the absorbed heat back to the surface (A). The latter further increases the surface's temperature, which results in an increase of surface radiation towards the atmosphere and an increase in the atmosphere's temperature. The continuous heat exchange between the surface and the atmosphere is known as the *greenhouse effect* and has a key role in controlling the planet's mean surface temperature, T. The relative abundance of GHG in the atmosphere regulates the amount of heat it absorbs. Therefore, changes in the concentration of GHG are followed by changes in T.

The EB task (Figure 1b) describes a simplified situation that begins with an energy balance at the surface; that is, the surface absorbs heat at the same rate it releases it ($S + A = R$ in Figure 1a). Then, it is assumed that a unique and instantaneous pulse of carbon dioxide (CO_2) is released at $t = 0$ to increase its concentration in the atmosphere. The EB task focuses on CO_2 for it is one of the main drivers of global warming (IPCC, 2018). The instantaneous increase in CO_2 is followed by an instantaneous increase in A, the heat radiation from the atmosphere to the surface. Thus, at $t = 0$, the surface is absorbing heat at a higher rate than that at which it is releasing it ($S + A > R$ in Figure 1a). The surface then begins to warm up as time elapses (T increases as t increases) and to increase its heat radiation towards the atmosphere (R increases as t increases). The atmosphere also begins to warm up and to further increase its heat radiation back to the surface (A increases as t increases). This further warms the surface and further increases the surface heat radiation towards the atmosphere (T and R continue to increase as t increases). This energy feedback loop between the surface and the atmosphere allows for R to increase enough so that a new energy balance is reached ($S + A = R$). This balance is accompanied by a new (higher) value of T. Thus, the goal of the EB task was to engage PSTs into thinking about how the Earth's energy balance's response to an increase in CO_2 results in an increase in the mean surface temperature, T, thus connecting CO_2 pollution to global warming.

The current paper examines the PSTs' reasoning regarding the feedback loop between the surface and the atmosphere in terms of a covariation between R and A with respect to time. The results will mainly focus on the PSTs' responses to the second part of the EB task, in which they were asked to draw the graph of T as a function of time t (Figure 1b).

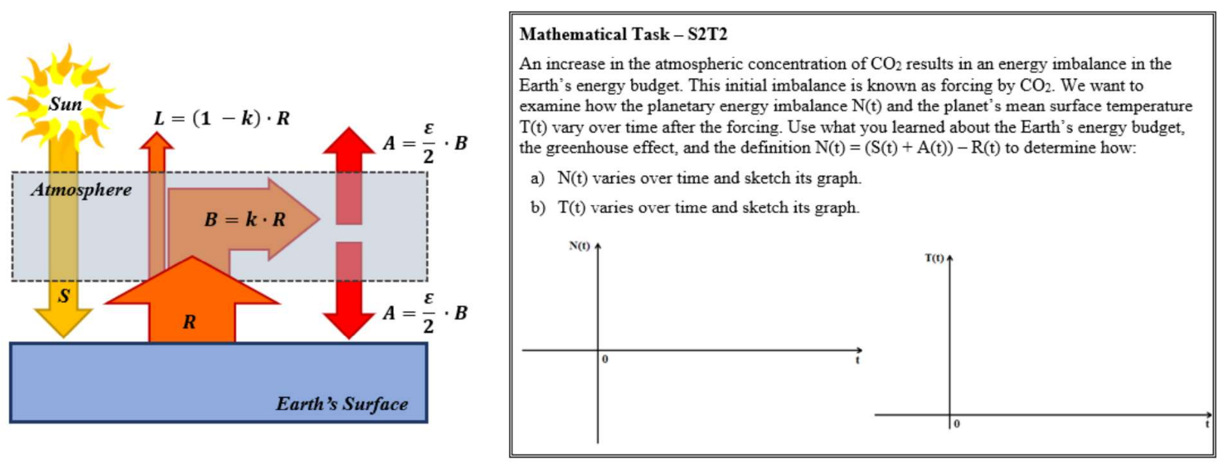

Figure 1: (a) An Earth's energy balance model (left) and (b) the EB task (right)

Data Collection

Before working on the EB task, the PSTs participated in a 32-minute-long, individual minilesson where some basic concepts related to the Earth's energy balance and the greenhouse effect were discussed. The goal was to provide PSTs with enough knowledge about those concepts so that they could start working on the EB task. The minilesson began with a 7-minute-long video retrieved from the NASA YouTube channel *NASAEarthObservatory* introducing the energy balance and the greenhouse effect. Then, each PST and I held a 5-minute-long Q&A session in which we clarified questions about the ideas discussed in the video. During the next 20 minutes, each PST and I worked with a diagram of the energy balance similar to the one in

Figure 1a. We talked about what each of the quantities S, R, L, B, and A represented and how they related to one another. Then, we talked about the energy balance at the surface as the equality $S + A = R$, and I illustrated it for some specific initial values of S, A, and R. We also discussed what inequalities such as $S + A > R$ or $S + A < R$ could mean in terms of temperature.

A week after the minilesson, each PST completed the EB task during a 60-minute-long, individual, task-based interview (Goldin, 2000). The interviews were semi-structured and had an interview protocol with pre-defined questions so that all participants received similar prompts during the interviews. To help PSTs understand how the energy balance responds to the increase in CO_2, the following recursive rules were made available to them: $B_t = k \cdot R_t$, $A_t = ½ \cdot B_t$, and $R_{t+1} = S + A_t$, for some $0 \le k \le 1$. By using these rules, they could find the values of B, A, and R for successive values of time, thus observing how these quantities change dynamically. This modelling strategy is known as *Discrete Event Simulation* and can be used to help students understand the dynamics of systems in particular situations (Hoad & Kunc, 2018).

Data Analysis

The interview videos and transcripts were analyzed through *thematic analysis* (Braun & Clarke, 2006; 2012). This qualitatively method of data analysis allows for systematically identifying, organizing, and offering insight into patterns of meaning across a data set though the development of codes and themes. Thematic analysis is a widely used analytic strategy to identify and make sense of collective or shared meanings and experiences. The method can be summarized into six phases: familiarizing yourself with your data, generating initial codes, searching for themes, reviewing themes, defining and naming, and producing the report.

I watched all interview videos and took notes while doing so. The videos were separated into shorter, more manageable *episodes*, each one covering a single topic or showing evidence of a particular way of reason covariationally. The notes informed my first round of coding for the interview transcripts. Then, the episode transcripts were sorted according to similar codes to look for patterns in participants' responses. This allowed me to revise and refine the initial codes, reducing them to two main themes. Thus, the five initial codes "asynchronous", "synchronous", "feedforward but not feedback", "verbalizing/indicating circularity", and "circular relationship" were collapsed into two themes: "Simultaneous Change", with "asynchronous" being the absence thereof and "synchronous" being the presence thereof, and "Circular causality", with "feedforward but not feedback" representing the absence thereof and "verbalizing/indicating circularity" and "circular relationship" representing the presence thereof.

Using the analytic framework previously described (Table 1), I indexed all episode transcripts into three analytic matrices, one per participant. These matrices allowed me to look for patterns in the distribution of themes, which provided the information needed to meet the research goals.

Results

The analysis revealed that, for this group of preservice teachers, two main cognitive realizations appear important to conceptualize the energy feedback loop between the surface and the atmosphere (the greenhouse effect) as a CLC between the quantities R and A: conceiving *simultaneity* of change and a *circular causality* relationship between those quantities. When one of these realizations was not supported, the PSTs developed inaccurate conceptualizations of the greenhouse effect and, by extension, this had an impact on their understanding of the link between CO_2 pollution and global warming.

Table 1: Analytic Framework

Theme	Descriptions
Simultaneity	<u>*Asynchronous*</u> The PST describes or represents changes in *A* and *R* as occurring asynchronously (*A* changes first, then *R* changes, then *A* again, and so on). <u>*Synchronous*</u> The PST describes or represents changes in *A* and *R* as occurring simultaneously as time elapses.
Causality	<u>*Linear*</u> The PST describes or represents causality in one direction between *A* and *R*. Either change in *A* causes change in *R* or change in *R* causes change in *A*. <u>*Circular*</u> The PST describes or represents a circular causality between *A* and *R* so that change in *A* causes change in *R*, which in turn causes change in *A* again.

The PSTs explored the greenhouse effect as an energy feedback loop while working on their graphs of $T = g(t)$. I start the discussion with Kris' work because she conceptualized the greenhouse effect in terms of CLC. More specifically, Kris made remarks about *R* and *A* increasing *simultaneously* as time, *t*, increased: "So, as *R* increases, *A* increases … the new *R* is affected by *S* plus *A* [*points at S and A*]. So, when *A* increases, *R* is going to be [bigger]. It can't just keep increasing". Kris also made remarks suggesting she conceived of a *circular causality* relationship between the quantities *R* and *A*.

> Well, [the surface] keeps in taking. I think it is warming up because once we added more CO_2, that is less of the emitted energy that is getting just like shut out passed the atmosphere, leaked from it. So then, more of it is going to be absorbed by the atmosphere … Whatever is absorbed by the atmosphere [*points at B*] is going to be absorbed back into the [*points at the surface*], well half of that plus the sun's energy [*points at S*] is going to be absorbed by the Earth, which is going to keep increasing, as we saw with like the 400 [*points at the R-value of "400"*]. Then, from the *A* value [*with a capped marker, traces the top half of a circle, going from R to A*], just with the *A*, [the surface] absorbs 160, and then we add a new *R*-value [*with the capped marker, traces the bottom half of the circle, going from A to R*], whatever that was, and then [the surface] absorbs 164 [*with the capped marker, re-traces the top half of the circle, going from R to A*]. So, I think it is going to keep increasing [*draws an increasing, concave-downward graph for T = g(t) that appears to have a horizontal asymptote that she labels as "new equilibrium temperature"*].

In the above excerpt, Kris repeatedly referred to the relationship between *R* and *A* as a "cycle", which suggests an awareness of circular causality between the quantities. Also, notice how she gestured both, the feedforward relationship from *A* to *R* and the feedback relationship from *R* back to *A*, further suggesting circular causality. Kris also drew an accurate graph of $T = g(t)$ showing asymptotic growth towards a new equilibrium value (Figure 2a); this suggests an awareness of the balancing quality of the feedback loop. Kris's CLC coincided with her demonstrating an accurate conception of the greenhouse effect, which guided her to conclude that an increase in CO_2 causes a warming effect over the planet's surface, correctly relating CO_2 pollution to global warming.

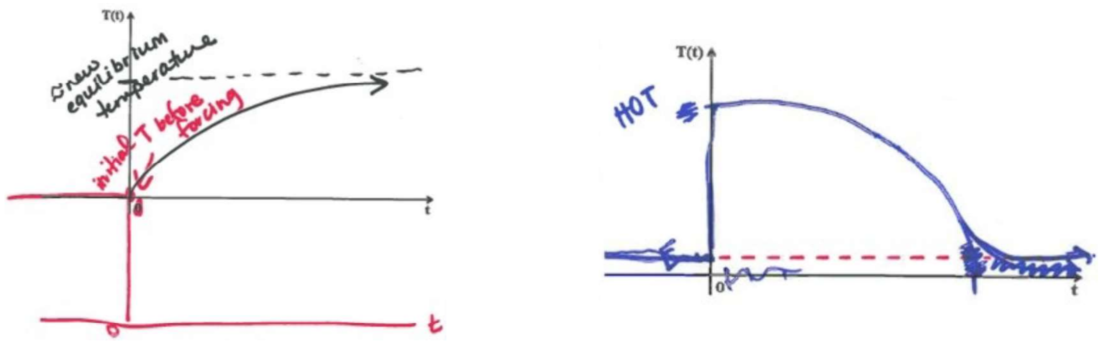

Figure 2: (a) Kris's graph of *T* = *g*(*t*) (left) and (b) Pam's graph of *T* = *g*(*t*) (right)

The case of Pam illustrates the conceptualization of OLC, where simultaneous change is conceived but not circular causality. She described *R* and *A* as two quantities increasing in tandem as *t* increased but only in the *A*-to-*R* direction ("*R* increases <u>*as*</u> [emphasis added] *A* increases. So, <u>*as*</u> [emphasis added] our *A* increases, *R* increases"). Pam also interpreted the increase in *R* and *A* as the planet's surface emitting more heat than the heat it was absorbing, which suggested additional evidence of only conceiving causality from *A* to *R*.

> A lot [of heat] is going in, but more is coming out, like *R* increases as *A* increases. So, as our *A* increases, *R* increases. But our *S* is staying the same. But our *A* is always less than *R*. So, more [heat] is coming out [*pauses to think*]. So, the Earth is trying to cool itself off, so the temperature is decreasing from here to here.

Pam again implied that an increase in *A* causes an increase in *R*, but did not appear aware that the increase in *R* also causes a new increase in *A*. She referred to *R* as heat leaving the surface and being larger than the heat absorbed by it. This claim overlooked that *A* is actually a fraction of *R* that is reabsorbed by the surface. This might have kept Pam from conceptualizing the feedback relationship from *R* back to *A*. Pam's OLC coincided with her demonstrating an inaccurate conception of the greenhouse effect, which may have led her to incorrectly conclude that the planet's surface cools down after an increase in CO_2, as her graph shows (Figure 2b).

Finally, Jodi's reasoning did not support simultaneous change but supported circular causality between *R* and *A*. In the following excerpt, Jodi appeared to imagine *R* and *A* as changing *asynchronously*—*A* changes first, then *R* changes, and then *A* again, and so on—but seemed aware of the circular causality between those two quantities.

> I am trying to look at the differences. So, like here [*points at the R-value "390" and the A-value "150"*]. Ok, so here the change was five [*points at the R-value "395" and the A-value "155"*]. The change was two [*points at the R-value "397" and the A-value "157"*]. Is it, I mean, is it not changing? … The flow of energy increased by five [*points at the A-value "155"*], but then it decreased by five [*points at the R-value "395"*]. Then it increased by two [*points at the A-value "157"*], and then it decreased by two [*points at the R-value "397"*]. So, it is almost as if there was no change in temperature because I associate like energy as kind of having a relationship with temperature. So, if the energy increases, then the temperature increases. But, in this scenario, an equal change in energy [*points at A*] was an equal change in output [*with her index finger, traces the bottom half of a circle from A to R*]

> … Ok, cycle started here [*points at B*], and here the Earth's temperature would've been something. Equal input of energy, equal output of energy [*with her index finger, traces a circle connecting A, R and B*]. Ok. So, when the cycle started, there was an input of energy [*points at A*], and then it got released [*with her index finger, traces the bottom half of a circle from A to R*]. Another cycle starts [*points at B*], input of energy, release of energy [*with her index finger, traces a circle connecting B, A, and R*]. So, it would almost be like [*draws a periodic curve formed by identical arcs*].

Notice how Jodi described the changes in *R* and *A* as happening at different times rather than simultaneously. This suggests Jodi did not develop a multiplicative object and her reasoning may not be considered covariational. In contrast, she did allude to circular causality by tracing circles with her finger connecting *R* and *A*. I also interpreted the periodicity of her graphs (Figure 3) as additional evidence of awareness of circular causality. Jodi's way of reasoning coincided with her demonstrating an inaccurate conception of the greenhouse effect and arriving to an incorrect conclusion regarding the real impact of CO_2 pollution on the planet's surface temperature.

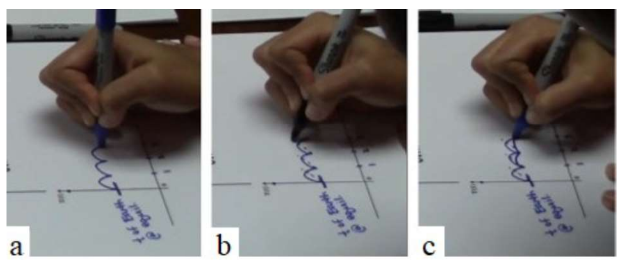

Figure 3: Jodi's periodic graphs of $T = g(t)$

Conclusion

Some situations that involve two quantities changing together over time also require the recognition of an underlying feedback loop structure between them. The study's results suggest that, for this group of preservice teachers, reasoning about feedback loop between quantities involved the ability to conceive *closed-loop covariation*, which in this study was characterized by two cognitive realizations: (i) the conception of *simultaneous change* and (ii) the recognition of *circular causality*. The first realization is based on the mental construction of a multiplicative object between those quantities (Saldanha & Thompson, 1998; Thompson et al., 2017), while the second realization involves noticing that changes in a quantity cause change in the second quantity, which in turn cause new changes in the first quantity and so on. The results also suggests that these two realizations, at least for these preservice teachers, appeared to be independent from each other. Kris demonstrated both realizations, while Pam and Jodi demonstrated one or the other but not both. An implication of this is that instructional strategies aiming to support students' ability to understand feedback loops mathematically should focus on developing both realizations at the same time.

It also important to point out that many school mathematical tasks involving change between quantities may only require *open-loop covariation*, where students oftentimes only need to attend to linear causality between the quantities. Examples of these are situations modeled by linear or quadratic functions. However, closed-loop covariation may be an interesting and novel way to explore situations involving exponential growth (e.g., compound interest or population growth), where the current value of the dependent quantity plays a role on how that quantity changes.

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